

# Marker-directed Optimization of UnCAL Graph Transformations

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**Abstract.** Buneman et al. proposed a graph algebra called UnCAL (Unstructured CALculus) for compositional graph transformations based on structural recursion, and we have recently applied to model transformations. The compositional nature of the algebra greatly enhances the modularity of transformations. However, intermediate results generated between composed transformations cause overhead. Buneman et al. proposed fusion rules that eliminate the intermediate results, but auxiliary rewriting rules that enable the actual application of the fusion rules are not apparent so far. UnCAL graph model includes the concept of markers, which correspond to recursive function call in the structural recursion. We have found that there are many optimization opportunities at rewriting level based on static analysis, especially focusing on markers. The analysis can safely eliminate redundant function calls. Performance evaluation shows its practical effectiveness for non-trivial examples in model transformations.

**Keywords:** program transformations, graph transformations, UnCAL

## 1 Introduction

Graph transformation has been an active research topic [9] and plays an important role in model-driven engineering [5, 11]; models such as UML diagrams are represented as graphs, and model transformation is essentially graph transformation. We have recently proposed a bidirectional graph transformation framework [6] based on providing bidirectional semantics to an existing graph transformation language UnCAL [4], and applied it to bidirectional model transformation by translating from existing model transformation language to UnCAL [10]. Our success in providing well-behaved bidirectional transformation framework was due to structural recursion in UnCAL, which is a powerful mechanism of visiting and transforming graphs. Transformation based on structural recursion

is inherently compositional, thus facilitates modular model transformation programming.

However, compositional programming may lead to many unnecessary intermediate results, which would make a graph transformation program terribly inefficient. As actively studied in programming language community, optimization like fusion transformation [12] is desired to make it practically useful. Despite a lot of work being devoted to fusion transformation of programs manipulating lists and trees, little work has been done on fusion on programs manipulating graphs. Although the original UnCAL has provided some fusion rules and rewriting rules to optimize graph transformations [4], we believe that further work and enhancement on fusion and rewriting are required.

The key idea presented in this paper is to analyze input/output markers, which are sort of labels on specific set of nodes in the UnCAL graph model and are used to compose graphs by connecting nodes with matching input and output markers. By statically analyzing connectivity of UnCAL by our marker analysis, we can simplify existing fusion rule. Consider, for instance, the following existing generic fusion rule of the structural recursion in UnCAL:

$$\begin{aligned} & \mathbf{rec}(\lambda(\$l_2, \$t_2).e_2)(\mathbf{rec}(\lambda(\$l_1, \$t_1).e_1)(e_0)) \\ & = \mathbf{rec}(\lambda(\$l_1, \$t_1).\mathbf{rec}(\lambda(\$l_2, \$t_2).e_2)(e_1 @ \mathbf{rec}(\lambda(\$l_1, \$t_1).e_1)(\$t_1)))(e_0) \end{aligned}$$

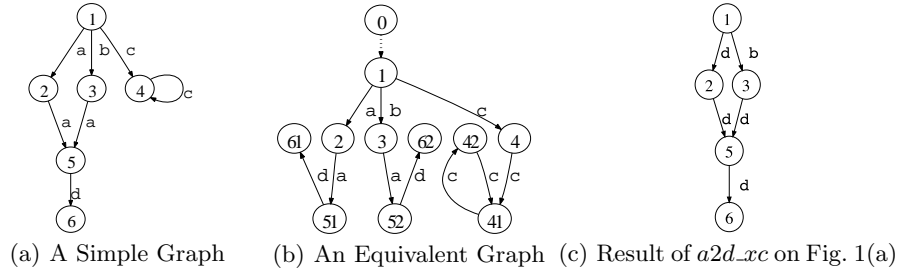
where  $\mathbf{rec}(\lambda(\$l, \$t).e)$  encodes a structural recursive function which is an important computation pattern and will be explained later. The graph constructor  $@$  connects two graphs by matching markers on nodes, and in this case, result of transformation  $e_1$  is combined to another structural recursion  $\mathbf{rec}(\lambda(\$l_1, \$t_1).e_1)$ . If we know by static analysis that  $e_1$  creates no output markers, or equivalently,  $\mathbf{rec}(\lambda(\$l_1, \$t_1).e_1)$  makes no recursive function call, then we can eliminate  $@ \mathbf{rec}(\lambda(\$l_1, \$t_1).e_1)(\$t_1)$  and further simplify the fusion rule. Our preliminary performance analysis reports relatively good evidence of usefulness of this optimization.

The main technical contributions of this paper are two folds: a sound and refined static inference of markers and a set of powerful rewriting rules for optimization using inferred markers. All have been implemented and tested with graph transformations widely recognized in software engineering research. The source code of the implementation can be downloaded via our project web site at [www.biglab.org](http://www.biglab.org).

The rest of this paper is organized as follows. Section 2 reviews UnCAL graph model, graph transformation language and existing optimizations. Section 3 proposes enhanced static analysis of markers. In Section 4, we build enhanced rewriting optimization algorithm based on the static analysis. Section 5 reports preliminary performance results. Section 6 reviews related work, and Section 7 concludes this paper.

## 2 UnCAL Graph Algebra and Prior Optimizations

In this section, we review the UnCAL graph algebra [3, 4], in which our graph transformation is specified.



**Fig. 1.** Graph Equivalence Based on Bisimulation

## 2.1 Graph Data Model

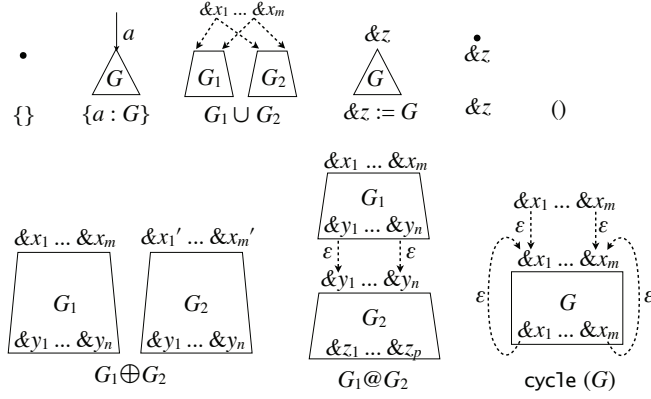
We deal with rooted, directed, and edge-labeled graphs with no order on outgoing edges. UnCAL graph data model has two prominent features, *markers* and  $\varepsilon$ -*edges*. Nodes may be marked with *input* and *output markers*, which are used as an interface to connect them to other graphs. An  $\varepsilon$ -edge represents a shortcut of two nodes, working like the  $\varepsilon$ -transition in an automaton. We use *Label* to denote the set of labels and  $\mathcal{M}$  to denote the set of markers.

Formally, a graph  $G$ , is a quadruple  $(V, E, I, O)$ , where  $V$  is a set of nodes,  $E \subseteq V \times (\text{Label} \cup \{\varepsilon\}) \times V$  is a set of edges,  $I \subseteq \mathcal{M} \times V$  is a set of pairs of an input marker and the corresponding node, and  $O \subseteq V \times \mathcal{M}$  is a set of pairs of nodes and associated output markers. For each marker  $\&x \in \mathcal{M}$ , there is at most one node  $v$  such that  $(\&x, v) \in I$ . The node  $v$  is called an *input node* with marker  $\&x$  and is denoted by  $I(\&x)$ . Unlike input markers, more than one node can be marked with an identical output marker. They are called *output nodes*. Intuitively, input nodes are root nodes of the graph (we allow a graph to have multiple root nodes, and for singly rooted graphs, we often use default marker  $\&$  to indicate the root), while an output node can be seen as a “context-hole” of graphs where an input node with the same marker will be plugged later. We write  $\text{inMarker}(G)$  to denote the set of input markers and  $\text{outMarker}(G)$  to denote the set of output markers in a graph  $G$ .

Note that multiple-marker graphs are meant to be an internal data structure for graph composition. In fact, the initial source graphs of our transformation have one input marker (single-rooted) and no output markers (no holes). For instance, the graph in Fig. 1(a) is denoted by  $(V, E, I, O)$  where  $V = \{1, 2, 3, 4, 5, 6\}$ ,  $E = \{(1, a, 2), (1, b, 3), (1, c, 4), (2, a, 5), (3, a, 5), (4, c, 4), (5, d, 6)\}$ ,  $I = \{(\&, 1)\}$ , and  $O = \{\}$ .  $DB_{\mathcal{Y}}^{\mathcal{X}}$  denotes graphs with sets of input markers  $\mathcal{X}$  and output markers  $\mathcal{Y}$ .  $DB_{\mathcal{Y}}^{\{\&\}}$  is abbreviated to  $DB_{\mathcal{Y}}$ .

## 2.2 Notion of Graph Equivalence

Two graphs are value equivalent if they are bisimilar. Please refer to [4] for the complete definition. For instance, the graph in Fig. 1(b) is value equivalent to



**Fig. 2.** Graph Constructors

the graph in Fig. 1(a); the new graph has an additional  $\varepsilon$ -edge (denoted by the dotted line), duplicates the graph rooted at node 5, and unfolds and splits the cycle at node 4. Unreachable parts are also disregarded, i.e., two bisimilar graphs are still bisimilar if one adds subgraphs unreachable from input nodes.

This value equivalence provides optimization opportunities because we can rewrite transformation so that transformation before and after rewriting produce results that are bisimilar to each other [4]. For example, optimizer can freely cut off expressions that is statically determined to produce unreachable parts.

### 2.3 Graph Constructors

Figure 2 summarizes the nine graph constructors that are powerful enough to describe arbitrary (directed, edge-labeled, and rooted) graphs [4]. Here,  $\{\}$  constructs a root-only graph,  $\{a : G\}$  constructs a graph by adding an edge with label  $a \in \text{Label} \cup \{\varepsilon\}$  pointing to the root of graph  $G$ , and  $G_1 \cup G_2$  adds two  $\varepsilon$ -edges from the new root to the roots of  $G_1$  and  $G_2$ . Also,  $\&xx := G$  associates an input marker,  $\&xx$ , to the root node of  $G$ ,  $\&yy$  constructs a graph with a single node marked with one output marker  $\&yy$ , and  $()$  constructs an empty graph that has neither a node nor an edge. Further,  $G_1 \oplus G_2$  constructs a graph by using a componentwise ( $V, E, I$  and  $O$ ) union.  $\cup$  differs from  $\oplus$  in that  $\cup$  unifies input nodes while  $\oplus$  does not.  $\oplus$  requires input markers of operands to be disjoint, while  $\cup$  requires them to be identical.  $G_1 @ G_2$  composes two graphs vertically by connecting the output nodes of  $G_1$  with the corresponding input nodes of  $G_2$  with  $\varepsilon$ -edges, and  $\text{cycle}(G)$  connects the output nodes with the input nodes of  $G$  to form cycles. Newly created nodes have unique identifiers. They are, together with other operators, bisimulation generic [4], i.e., bisimilar result is obtained for bisimilar operands.

$$\begin{array}{l}
e ::= \{\} \mid \{l : e\} \mid e \cup e \mid \&x := e \mid \&y \mid () \\
\mid e \oplus e \mid e @ e \mid \mathbf{cycle}(e) & \{ \text{constructor} \} \\
\mid \$g & \{ \text{graph variable} \} \\
\mid \mathbf{let} \$g = e \mathbf{in} e & \{ \text{variable binding} \} \\
\mid \mathbf{if} l = l \mathbf{then} e \mathbf{else} e & \{ \text{conditional} \} \\
\mid \mathbf{rec}(\lambda(\$l, \$g).e)(e) & \{ \text{structural recursion application} \} \\
l ::= a \mid \$l & \{ \text{label } (a \in \text{Label}) \text{ and label variable} \}
\end{array}$$

**Fig. 3.** Core UnCAL Language

*Example 1.* The graph equivalent to that in Fig. 1(a) can be constructed as follows (though not uniquely).

$$\begin{aligned}
&\&z @ \mathbf{cycle}((\&z := \{\mathbf{a} : \{\mathbf{a} : \&z_1\}\} \cup \{\mathbf{b} : \{\mathbf{a} : \&z_1\}\} \cup \{\mathbf{c} : \&z_2\}) \\
&\quad \oplus (\&z_1 := \{\mathbf{d} : \{\}\}) \\
&\quad \oplus (\&z_2 := \{\mathbf{c} : \&z_2\})) \quad \square
\end{aligned}$$

For simplicity, we often write  $\{a_1 : G_1, \dots, a_n : G_n\}$  to denote  $\{a_1 : G_1\} \cup \dots \cup \{a_n : G_n\}$ , and  $(G_1, \dots, G_n)$  to denote  $(G_1 \oplus \dots \oplus G_n)$ .

## 2.4 UnCAL Syntax

UnCAL (Unstructured Calculus) is an internal graph algebra for the graph query language UnQL, and its core syntax is depicted in Fig. 3. It consists of the graph constructors, variables, variable bindings, conditionals, and structural recursion. We have already detailed the data constructors, while variables, variable bindings and conditionals are self explanatory. Therefore, we will focus on *structural recursion*, which is a powerful mechanism in UnCAL to describe graph transformations.

A function  $f$  on graphs is called a structural recursion if it is defined by the following equations

$$\begin{aligned}
f(\{\}) &= \{\} \\
f(\{\$l : \$g\}) &= e @ f(\$g) \\
f(\$g_1 \cup \$g_2) &= f(\$g_1) \cup f(\$g_2),
\end{aligned}$$

and  $f$  can be encoded by  $\mathbf{rec}(\lambda(\$l, \$g).e)$ . Despite its simplicity, the core UnCAL is powerful enough to describe interesting graph transformation including all graph queries (in UnQL) [4], and nontrivial model transformations [8].

*Example 2.* The following structural recursion  $a2d\_xc$  replaces all labels  $\mathbf{a}$  with  $\mathbf{d}$  and removes edges labeled  $\mathbf{c}$ .

$$\begin{aligned}
a2d\_xc(\$db) = \mathbf{rec}(\lambda(\$l, \$g). \mathbf{if} \$l = \mathbf{a} \mathbf{then} \quad \{\mathbf{d} : \&\} \\
\quad \mathbf{else if} \$l = \mathbf{c} \mathbf{then} \quad \{\varepsilon : \&\} \\
\quad \mathbf{else} \quad \quad \quad \{\$l : \&\}) (\$db)
\end{aligned}$$

The nested **ifs** correspond to  $e$  in the above equations. Applying the function  $a2d\_xc$  to the graph in Fig. 1(a) yields the graph in Fig. 1(c).  $\square$



$$\begin{array}{l}
& \&x := (\&z := e) \longrightarrow \&x.\&z := e \quad \&x := (e_1 \oplus e_2) \longrightarrow (\&x := e_1) \oplus (\&x := e_2) \\
& e \cup \{\} \longrightarrow e \quad \{\} \cup e \longrightarrow e \quad e \oplus () \longrightarrow e \quad () \oplus e \longrightarrow e \\
& () @ e \longrightarrow () \quad \frac{e :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \mathcal{X} \cap \mathcal{Y} = \phi}{\mathbf{cycle}(e) \longrightarrow e}
\end{array}$$

Fig. 4. Auxiliary Rewriting Rules

## 2.7 Other Prior Rewriting Rules

Apart from fusion rules, the following rewriting rules for **rec** are proposed in [4] for optimizations. Type of  $e$  is assumed to be  $DB_{\mathcal{Z}}^{\mathcal{Z}}$ . They simplify the argument of **rec** and increases chances of fusions. Some of them are recapped below.

$$\begin{array}{l}
\mathbf{rec}(\lambda(\$l, \$t).e)(\{\}) \quad = {}^1 \bigoplus_{\&z \in \mathcal{Z}} \&z := \{\} \\
\mathbf{rec}(\lambda(\$l, \$t).e)(\{l : d\}) = e[l/\$l][d/\$t] @ \mathbf{rec}(\lambda(\$l, \$t).e)(d)
\end{array}$$

The first rule eliminates **rec**, while the second rule eliminates an edge from the argument.

Additional rules proposed by (full version of) Hidaka et al. [8] to further simplify the body of **rec** are given in Fig. 4. The rules in the last line in Fig. 4 can be generalized by static analysis of the marker in the following section. And given the static analysis, we can optimize further as described in Section 4.

## 3 Enhanced Static Analysis

This section proposes our enhanced marker analysis. Figure 5 shows the proposed marker *inference* rules for UnCAL. Static environment  $\Gamma$  denotes mapping from variables to their types. We assume that the types of free variables are given. Since we focus on graph values, we omit rules for labels. Roughly speaking,  $DB_{\mathcal{Y}}^{\mathcal{X}}$  is a type for graphs that have  $\mathcal{X}$  input markers exactly and have at most  $\mathcal{Y}$  output markers, which will be shown formally by Lemma 1.

The original typing rules were provided based on the subtyping rule

$$\frac{\Gamma \vdash e :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \mathcal{Y} \subseteq \mathcal{Y}'}{\Gamma \vdash e :: DB_{\mathcal{Y}'}^{\mathcal{X}}}$$

and required the arguments of  $\cup$ ,  $\oplus$ , **if** to have identical sets of output markers. Unlike the original rules, the proposed type system does not use the subtyping rule directly for inference. Combined with the forward evaluation semantics  $\mathcal{F}[\![\ ]\!]_{\text{fwd}}$  that is summarized in [6], we have the following type safety property.

<sup>1</sup> Original right hand side was  $\{\}$  in [4], but we corrected here.

<sup>5</sup> Original rule (let's say  $@_o$ ) which requires  $\mathcal{Y}_1 = \mathcal{X}_2$  is relaxed here. Our  $@$  can be defined by  $g_1 @ g_2 = (g_1 @_o \text{ld}_{\mathcal{X}_2 \setminus \mathcal{Y}_1}^{\mathcal{X}_2 \setminus \mathcal{Y}_1}) @_o (\mathbf{Bot}_0^{\mathcal{Y}_1 \setminus \mathcal{X}_2} \oplus g_2)$ . This particular definition in which markers  $\mathcal{Y}_1 \setminus \mathcal{X}_2$  are peeled off is close to the original semantics because final output markers coincide. Extension in which these excess output markers remain would be possible, allowing the markers to be used later to connect to other graphs.

$$\begin{aligned}
\mathcal{X} \cdot \mathcal{Y} &\stackrel{\text{def}}{=} \{\&x \cdot \&y \mid \&x \in \mathcal{X}, \&y \in \mathcal{Y}\} \quad \& \cdot \&x = \&x \cdot \& = \&x \quad (\&x \cdot \&y) \cdot \&z = \&x \cdot (\&y \cdot \&z) \\
&\frac{\Gamma \vdash l :: \text{Label} \quad \Gamma \vdash e_1 :: DB_{\mathcal{Y}_1}^{\mathcal{X}}}{\Gamma \vdash \{l : e\} :: DB_{\mathcal{Y}}^{\mathcal{X}}} \quad \frac{\Gamma \vdash e_1 :: DB_{\mathcal{Y}_1}^{\mathcal{X}} \quad \Gamma \vdash e_2 :: DB_{\mathcal{Y}_2}^{\mathcal{X}}}{\Gamma \vdash e_1 \cup e_2 :: DB_{\mathcal{Y}_1 \cup \mathcal{Y}_2}^{\mathcal{X}}} \\
&\frac{}{\Gamma \vdash \{\} :: DB_{\emptyset}} \quad \frac{\Gamma \vdash e :: DB_{\mathcal{Y}}^{\mathcal{Z}}}{\Gamma \vdash \&x := e :: DB_{\mathcal{Y}}^{\{\&x\} \cdot \mathcal{Z}}} \quad \frac{}{\Gamma \vdash \&y :: DB_{\{\&y\}}} \\
&\frac{\Gamma \vdash e_1 :: DB_{\mathcal{Y}_1}^{\mathcal{X}_1} \quad \Gamma \vdash e_2 :: DB_{\mathcal{Y}_2}^{\mathcal{X}_2} \quad \mathcal{X}_1 \cap \mathcal{X}_2 = \emptyset}{\Gamma \vdash e_1 \oplus e_2 :: DB_{\mathcal{Y}_1 \cup \mathcal{Y}_2}^{\mathcal{X}_1 \cup \mathcal{X}_2}} \quad \frac{\Gamma \vdash e_1 :: DB_{\mathcal{Y}_1}^{\mathcal{X}_1} \quad \Gamma \vdash e_2 :: DB_{\mathcal{Y}_2}^{\mathcal{X}_2}}{\Gamma \vdash e_1 @ e_2 :: DB_{\mathcal{Y}_2}^{\mathcal{X}_1}} \quad \frac{\Gamma \vdash e :: DB_{\mathcal{Y}}^{\mathcal{X}}}{\Gamma \vdash \mathbf{cycle}(e) :: DB_{\mathcal{Y} \setminus \mathcal{X}}^{\mathcal{X}}} \\
&\frac{\Gamma \vdash e_a :: DB_{\mathcal{Y}}^{\mathcal{X}}}{\Gamma \{\$l \mapsto \text{Label}, \$g \mapsto DB_{\mathcal{Y}}\} \vdash e_b :: DB_{\mathcal{Z}_o}^{\mathcal{Z}_i} \quad \mathcal{Z} = \mathcal{Z}_i \cup \mathcal{Z}_o} \quad \frac{\Gamma \{\$g\} = DB_{\mathcal{Y}}^{\mathcal{X}}}{\Gamma \vdash \$g :: DB_{\mathcal{Y}}^{\mathcal{X}}} \\
&\frac{\Gamma \vdash l_1 :: \text{Label} \quad \Gamma \vdash l_2 :: \text{Label} \quad \Gamma \vdash e_t :: DB_{\mathcal{Y}_t}^{\mathcal{X}} \quad \Gamma \vdash e_f :: DB_{\mathcal{Y}_f}^{\mathcal{X}}}{\Gamma \vdash \mathbf{if} \, l_1 = l_2 \, \mathbf{then} \, e_t \, \mathbf{else} \, e_f :: DB_{\mathcal{Y}_t \cup \mathcal{Y}_f}^{\mathcal{X}}} \quad \frac{\Gamma \vdash e_1 :: DB_{\mathcal{Y}_1}^{\mathcal{X}_1} \quad \Gamma \{\$g \mapsto DB_{\mathcal{Y}_1}^{\mathcal{X}_1}\} \vdash e_2 :: DB_{\mathcal{Y}_2}^{\mathcal{X}_2}}{\Gamma \vdash \mathbf{let} \, \$g = e_1 \, \mathbf{in} \, e_2 :: DB_{\mathcal{Y}_2}^{\mathcal{X}_2}}
\end{aligned}$$

**Fig. 5.** UnCAL Static Typing (Marker Inference) Rules: Rules for *Label* are Omitted

**Lemma 1 (Type Safety).** *Assume that  $g$  is the graph obtained by  $g = \mathcal{F}[e]$  for an expression  $e$ . Then,  $\vdash e :: DB_{\mathcal{Y}}^{\mathcal{X}}$  implies both  $\text{inMarker}(g) = \mathcal{X}$  and  $\text{outMarker}(g) \subseteq \mathcal{Y}$ .*

Lemma 1 guarantees that the set of input markers estimated by the type inference is exact in the sense that the set of input markers generated by evaluation exactly coincides with that of the inferred type. For the output markers, the type system provides an over-approximation in the sense that the set of output markers generated by evaluation is a subset of the inferred set of output markers. Since the statement on the input marker is a direct consequence of the rules in [4], we focus that on the output markers and prove it. The proof, which is based on induction on the structure of UnCAL expression, is in the full version [7] of this paper.

## 4 Enhanced Rewiring Optimization

This section proposes enhanced rewriting optimization rules based on the static analysis shown in the previous section.

### 4.1 Rule for @ and Revised Fusion Rule

Statically-inferred markers enables us to optimize expressions much more. We can generalize the rewriting rules in the last row of Fig. 4 by not just referring to the pattern of subexpressions but its estimated markers, such as



$$() @ e \longrightarrow () \quad \Rightarrow \quad \frac{e_1 :: DB_{\emptyset}^{\mathcal{X}}}{e_1 @ e_2 \longrightarrow e_1} \quad (2)$$

As we have seen in Sect. 2, we have two fusion rules for **rec**. Although the first rule can be used to gain performance, the second rule is more complex so less performance gain is expected. Using (2), we can relax the condition of the first condition of the fusion rule (1) to increase chances to apply the first rule as follows.

$$\begin{aligned} & \mathbf{rec}(\lambda(\$l_2, \$t_2).e_2)(\mathbf{rec}(\lambda(\$l_1, \$t_1).e_1)(e_0)) \\ &= \mathbf{rec}(\lambda(\$l_1, \$t_1).\mathbf{rec}(\lambda(\$l_2, \$t_2).e_2)(e_1))(e_0) \\ & \quad \text{if } \$t_2 \text{ does not appear free in } e_2, \text{ or } \underline{e_1 :: DB_{\emptyset}^{\mathcal{X}}} \end{aligned}$$

Here, the underlined part is changed.

## 4.2 Further Optimization with Static Marker Information

In this section, general rules for  $e_1 @ e_2$  is investigated. First to eliminate  $@ e_2$ , and then to statically compute  $@$  by plugging  $e_2$  into  $e_1$ .

### 4.2.1 Static Output-Marker Removal Algorithm and Soundness

For more general cases of  $@$  where connections by  $\varepsilon$  do not happen, we have the following rule.

$$\frac{e_1 :: DB_{\mathcal{Y}_1}^{\mathcal{X}} \quad e_2 :: DB_{\mathcal{Z}}^{\mathcal{Y}_2} \quad \mathcal{Y}_1 \cap \mathcal{Y}_2 = \emptyset \quad \mathbf{Rm}_{\mathcal{Y}_1} \langle\langle e_1 \rangle\rangle = e}{e_1 @ e_2 \longrightarrow e}$$

$\mathbf{Rm}_{\mathcal{Y}} \langle\langle e \rangle\rangle$  denotes static removal of the set of output markers. Without this, rewriting result in spurious output markers from  $e_1$  remained in the final result. The formal definition of  $\mathbf{Rm}_{\mathcal{Y}} \langle\langle e \rangle\rangle$  is shown below.

$$\begin{aligned} & \mathbf{Rm}_{\emptyset} \langle\langle e \rangle\rangle = e \quad \mathbf{Rm}_{\mathcal{X} \cup \mathcal{Y}} \langle\langle e \rangle\rangle = \mathbf{Rm}_{\mathcal{Y}} \langle\langle \mathbf{Rm}_{\mathcal{X}} \langle\langle e \rangle\rangle \rangle \quad \mathbf{Rm}_{\{\&y\}} \langle\langle \{\} \rangle\rangle = \{\} \\ & \mathbf{Rm}_{\{\&y\}} \langle\langle () \rangle\rangle = () \quad \mathbf{Rm}_{\{\&y\}} \langle\langle \&y \rangle\rangle = \{\} \quad \mathbf{Rm}_{\{\&y\}} \langle\langle \&x \rangle\rangle = \&x \\ & \mathbf{Rm}_{\{\&y\}} \langle\langle e_1 \odot e_2 \rangle\rangle = \mathbf{Rm}_{\{\&y\}} \langle\langle e_1 \rangle\rangle \odot \mathbf{Rm}_{\{\&y\}} \langle\langle e_2 \rangle\rangle \quad (\odot \in \{\cup, \oplus\}) \\ & \mathbf{Rm}_{\{\&y\}} \langle\langle \&x := e \rangle\rangle = (\&x := \mathbf{Rm}_{\{\&y\}} \langle\langle e \rangle\rangle) \\ & \mathbf{Rm}_{\{\&y\}} \langle\langle \{l : e\} \rangle\rangle = \{l : \mathbf{Rm}_{\{\&y\}} \langle\langle e \rangle\rangle\} \\ & \mathbf{Rm}_{\{\&y\}} \langle\langle e_1 @ e_2 \rangle\rangle = e_1 @ \mathbf{Rm}_{\{\&y\}} \langle\langle e_2 \rangle\rangle \\ & \mathbf{Rm}_{\{\&y\}} \langle\langle \mathbf{if } b \text{ then } e_1 \text{ else } e_2 \rangle\rangle = \mathbf{if } b \text{ then } \mathbf{Rm}_{\{\&y\}} \langle\langle e_1 \rangle\rangle \text{ else } \mathbf{Rm}_{\{\&y\}} \langle\langle e_2 \rangle\rangle \end{aligned}$$

Since the output markers of the result of  $@$  are not affected by that of  $e_1$ ,  $e_1$  is not visited in the rule of  $@$ . In the following,  $\mathbf{ld}_{\mathcal{Y}}^{\mathcal{Y}}$  and  $\mathbf{Bot}_{\emptyset}^{\mathcal{Y}}$  are respectively defined as  $\bigoplus_{\&z \in \mathcal{Y}} \&z := \&z$  and  $\bigoplus_{\&z \in \mathcal{Y}} \&z := \{\}$ .

$$\frac{e :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \&y \in (\mathcal{Y} \setminus \mathcal{X}) \quad \mathbf{Rm}_{\{\&y\}} \langle\langle e \rangle\rangle = e'}{\mathbf{Rm}_{\{\&y\}} \langle\langle \mathbf{cycle}(e) \rangle\rangle = \mathbf{cycle}(e')} \quad \frac{e :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \&y \notin (\mathcal{Y} \setminus \mathcal{X})}{\mathbf{Rm}_{\{\&y\}} \langle\langle \mathbf{cycle}(e) \rangle\rangle = \mathbf{cycle}(e)}$$

$$\frac{\$v :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \&y \notin \mathcal{Y}}{\text{Rm}_{\{\&y\}}\langle\langle \$v \rangle\rangle = \$v} \quad \frac{\$v :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \&y \in \mathcal{Y}}{\text{Rm}_{\{\&y\}}\langle\langle \$v \rangle\rangle = \$v @ (\text{Bot}_{\emptyset}^{\{\&y\}} \oplus \text{Id}_{\mathcal{Y} \setminus \{\&y\}}^{\{\&y\}})}$$

The first rule of  $\$v$  says that according to the safety of type inference,  $\&y$  is guaranteed not to result at run-time, so the expression  $\$v$  remains unchanged. The second rule actually removes the output marker  $\&y_j$ , but static removal is impossible. So the removal is deferred till run-time. The output node marked  $\&y_j$  is connected to node produced by  $\&y := \{\}$ . Since the latter node has no output marker, the original output marker disappears from the graph produced by the evaluation. The rest of the  $\&y_k := \&y_k$  does no operation on the marker. Since estimation  $\mathcal{Y}$  is the upper bound, the output maker may not be produced at run-time. If it is the case, connection with  $\varepsilon$ -edge by  $@$  does not occur, and the nodes produced by the  $:=$  expressions are left unreachable, so the transformation is still valid. As another side effect,  $@$  may connect identically marked output nodes to single node. However, the graph before and after this “funneling” connection are bisimilar, since every leaf node with identical output markers are bisimilar by definition. Should the output nodes are to be further connected to other input nodes, the target node is always single, because more than one node with identical input marker is disallowed by the data model. So this connection does no harm. Note that the second rule increases the size of the expression, so it may increase the cost of evaluation.

$$\frac{\text{rec}(\lambda(\$l, \$t).e_b)(e_a) :: DB_{\mathcal{Y} \cdot \mathcal{Z}}^{\mathcal{X} \cdot \mathcal{Z}} \quad \&y \in \mathcal{Y} \quad \text{Rm}_{\{\&y\}}\langle\langle e_a \rangle\rangle = e'_a}{\text{Rm}_{\{\&y, \&z \mid \&z \in \mathcal{Z}\}}\langle\langle \text{rec}(\lambda(\$l, \$t).e_b)(e_a) \rangle\rangle = \text{rec}(\lambda(\$l, \$t).e_b)(e'_a)}$$

For  $\text{rec}$ , one output marker  $\&y$  in  $e_a$  corresponds to  $\{\&y\} \cdot \mathcal{Z} = \{\&y.\&z \mid \&z \in \mathcal{Z}\}$  in the result. So removal of  $\&y$  from  $e_a$  results in removal of all of the  $\{\&y\} \cdot \mathcal{Z}$ . So only removal of all of  $\{\&y.\&z \mid \&z \in \mathcal{Z}\}$  at a time is allowed.

**Lemma 2 (Soundness of Static Output-Marker Removal Algorithm).**  
*Assume that  $G = (V, E, I, O)$  is a graph obtained by  $G = \mathcal{F}[e]$  for an expression  $e$ , and  $e'$  is the expression obtained by  $\text{Rm}_{\mathcal{Y}}\langle\langle e \rangle\rangle$ . Then, we have  $\mathcal{F}[e'] = (V, E, I, \{(v, \&y) \in O \mid \&y \notin \mathcal{Y}\})$ .*

Lemma 2 guarantees that no output marker in  $\mathcal{Y}$  appears at run-time if  $\text{Rm}_{\mathcal{Y}}\langle\langle e \rangle\rangle$  is evaluated.

#### 4.2.2 Plugging Expression to Output Marker Expression

The following rewriting rule is to plug an expression into another through correspondingly marked node.

$$\{l : \&y\} @ (\&y := e) \longrightarrow \{l : e\}$$

This kind of rewriting was actually implicitly used in the exemplification of optimization in [4], but was not generalized. We can generalize this rewriting as

$$e @ (\&y := e') \longrightarrow \begin{cases} \text{Rm}_{\mathcal{Y} \setminus \{\&y\}}\langle\langle e \rangle\rangle[e'/\&y] & \text{if } \&y \in \mathcal{Y} \text{ where } e :: DB_{\mathcal{Y}}^{\mathcal{X}} \\ \text{Rm}_{\mathcal{Y}}\langle\langle e \rangle\rangle & \text{otherwise.} \end{cases}$$

where  $e[e'/\&y]$  denotes substitution of  $\&y$  by  $e'$  in  $e$ . Since nullary constructors  $\{\}$ ,  $()$ , and  $\&x \neq \&y$  do not produce output marker  $\&y$ , the substitution takes no effect and the rule in the latter case apply. So we focus on the former case in the sequel. For most of the constructors the substitution rules are rather straightforward:

$$\begin{aligned}
&\&y[e/\&y] = e \\
&(e_1 \odot e_2)[e/\&y] = (e_1[e/\&y]) \odot (e_2[e/\&y]) \quad (\odot \in \{\cup, \oplus\}) \\
&(\&x := e)[e'/\&y] = (\&x := (e[e'/\&y])) \\
&\{l : e\}[e'/\&y] = \{l : (e[e'/\&y])\} \\
&(e_1 @ e_2)[e/\&y] = e_1 @ (e_2[e/\&y]) \\
&(\mathbf{if} \ b \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2)[e/\&y] = \mathbf{if} \ b \ \mathbf{then} \ (e_1[e/\&y]) \ \mathbf{else} \ (e_2[e/\&y])
\end{aligned}$$

Since the final output marker for  $@$  is not affected by that of  $e_1$ ,  $e_1$  is not visited in the rule of  $@$ . For **cycle**, we should be careful to avoid capturing of marker.

$$\mathbf{cycle}(e)[e'/\&y] = \begin{cases} \mathbf{cycle}(e[e'/\&y]) & \text{if } (\mathcal{Y}' \cap \mathcal{X}) = \emptyset \text{ where } e :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad e' :: DB_{\mathcal{Y}'} \\ \overline{\mathbf{cycle}}(e)[e'/\&y] & \text{otherwise.} \end{cases}$$

The above rule says that if  $\&y$  will be a “free” marker in  $e$ , that is, the output markers in  $e'$ , namely  $\mathcal{Y}'$  will not be captured by **cycle**, then we can plug  $e'$  into output marker expression in  $e$ . If some of the output markers in  $\mathcal{Y}'$  are included in  $\mathcal{X}$ , then the renaming is necessary. As suggested in the full version of [3], markers in  $\mathcal{X}$  instead of those in  $\mathcal{Y}'$  should be renamed. And that renaming can be compensated outside of **cycle** as follows:

$$\overline{\mathbf{cycle}}(e) \stackrel{\text{def}}{=} \left( \bigoplus_{\&x \in \mathcal{X}} \&x := \&tmp_x \right) @ \mathbf{cycle}(e[\&tmp_{x_1}/\&x_1] \dots [\&tmp_{x_M}/\&x_M])$$

where  $\&x_1, \dots, \&x_M = \mathcal{X}$  are the markers to be renamed, and  $\mathcal{X}$  of  $e :: DB_{\mathcal{Y}}^{\mathcal{X}}$  is used. Note that in the renaming, not only output markers, but also input markers are renamed.  $\&tmp_{x_1}, \dots, \&tmp_{x_M}$  are corresponding fresh (temporary) markers. The left hand side of  $@$  recovers the original name of the markers. After renaming by **cycle**, no marker is captured anymore, so substitution is guaranteed to succeed. For variable reference and **rec**, static substitution is impossible. So we resort to the following generic “fall back” rule.

$$\frac{e \in \{\$v, \mathbf{rec}(-)(-)\} \quad e :: DB_{\mathcal{Y}}^{\mathcal{X}} \quad \mathcal{Y} = \{\&y_1, \dots, \&y_j, \dots, \&y_n\}}{e[e'/\&y_j] = e @ \left( \begin{array}{l} \&y_1 := \&y_1, \dots, \&y_{j-1} := \&y_{j-1}, \&y_j := e', \\ \&y_{j-1} := \&y_{j-1}, \dots, \&y_n := \&y_n \end{array} \right)}$$

The “fall back” rule is used for **rec** because unlike output marker removal algorithm, we can not just plug  $e$  into  $e_a$  since that will not plug  $e$  but  $\mathbf{rec}(\lambda(\$l, \$t).e_b)(e)$  in the result. We could have used the inverse  $\mathbf{rec}(\lambda(\$l, \$t).e_b)^{-1}$  to plug  $\mathbf{rec}(\lambda(\$l, \$t).e_b)^{-1}(e')$  instead, but the inverse does not always exist in general.

The overall rewriting is conducted by two mutually recursive functions as follows: driver function  $P$  first apply itself to subexpressions recursively, and then apply function  $F$  that implements  $\longrightarrow$  and other rewriting rules recursively such as fusions described in this paper, on the result of  $P$ .

With respect to proposed rewriting rules in this section, the following theorem hold.

**Theorem 1 (Soundness of Rewriting).** *If  $e \longrightarrow e'$ , then  $\mathcal{F}[[e]]$  is bisimilar to  $\mathcal{F}[[e']]$ .*

It can be proved by simple induction on the structure of UnCAL expressions, and omitted here.

*Example 4.* The following transformation that apply selection after *consecutive* in Example 3

$\mathbf{rec}(\lambda(\$l_1, \$g_1). \mathbf{if} \$l_1 = \mathbf{a} \mathbf{then} \{\$l_1 : \$g_1\} \mathbf{else} \{\}) (\mathit{consecutive}(\$db))$   
is rewritten as follows:

$$\begin{aligned}
& \{ \text{expand definition of } \mathit{consecutive} \text{ and apply 2nd fusion rule } \} \\
= & \mathbf{rec}(\lambda(\$l, \$g). \mathbf{rec}(\lambda(\$l_1, \$g_1). \mathbf{if} \$l_1 = \mathbf{a} \mathbf{then} \{\$l_1 : \$g_1\} \mathbf{else} \{\}) \\
& \quad (\mathbf{rec}(\lambda(\$l', \$g'). \mathbf{if} \$l = \$l' \mathbf{then} \{\mathbf{result} : \$g'\} \mathbf{else} \{\}) (\$g)) (\$g)) (\$db) \\
& \quad @ \mathbf{rec}(\lambda(\$l, \$g). \mathbf{rec}(\lambda(\$l', \$g'). \\
& \quad \quad \mathbf{if} \$l = \$l' \mathbf{then} \{\mathbf{result} : \$g'\} \mathbf{else} \{\}) (\$g)) (\$g)) (\$db) \\
& \{ (2) \} \\
= & \mathbf{rec}(\lambda(\$l, \$g). \mathbf{rec}(\lambda(\$l_1, \$g_1). \mathbf{if} \$l_1 = \mathbf{a} \mathbf{then} \{\$l_1 : \$g_1\} \mathbf{else} \{\}) \\
& \quad (\mathbf{rec}(\lambda(\$l', \$g'). \mathbf{if} \$l = \$l' \mathbf{then} \{\mathbf{result} : \$g'\} \mathbf{else} \{\}) (\$g)) (\$db) \\
& \quad \{ \text{2nd fusion rule, (2), } \mathbf{rec} \text{ rule for } \mathbf{if} \text{ and } \{l : d\}, \text{ static label comparison } \} \\
= & \mathbf{rec}(\lambda(\$l, \$g). \mathbf{rec}(\lambda(\$l', \$g'). \{\}) (\$g)) (\$db)
\end{aligned}$$

This example demonstrates the second fusion rule promotes to the first. The top level edges of the result of *consecutive* are always labeled **result** while the selection selects subgraphs under edges labeled **a**. So the result will always be empty, and correspondingly the body of **rec** in the final result is  $\{\}$ .

More examples can be found in the full version [7] of this paper.

## 5 Implementation and Performance Evaluation

This section reports preliminary performance evaluations.

### 5.1 Implementation in GRoundTram

All of the transformation in the paper are implemented in **GRoundTram**, or **Graph Roundtrip Transformation for Models**, which is a system to build a bidirectional transformation between two models (graphs). All the source codes are available online at [www.biglab.org](http://www.biglab.org). The following experimental results are obtained by the system.

**Table 1.** Summary of Experiments (running time is in CPU seconds)

	direction	no rewriting	previous [4, 8]	ours
<i>Class2RDB</i>	forward	1.18	0.68	0.68
	backward	14.5	7.99	7.89
<i>PIM2PSM</i>	forward	0.08	0.77 (2*3)	0.07 (2*13)
	backward	1.62	3.64	0.75
<i>C2Osel</i>	forward	0.04	0.04 (2*1)	0.05 (2*11)
	backward	2.26	0.26	0.27
<i>C2Osel'</i>	forward	0.05	0.06 (2*1)	0.04 (2*11)
	backward	2.53	2.58	1.26
<i>UnQL</i>	forward	0.022	0.016 (1*1)	0.010 (1*1)
	backward	0.85	0.30	0.15

## 5.2 Performance Results

Performance evaluation was conducted on **GRoundTram**, running on MacOSX over MacBookPro 17 inch, with 3.06 GHz Intel Core 2 Duo CPU. Time complexity is PTIME for the size of input graph[4], and exponential in the size (number of compositions or nesting of **recs**) of the transformation. In the experiments, the size of input data (graph) is not very large (up to a hundred of nodes).

Table 1 shows the experimental results. Each running time includes time for forward and backward transformations [6], and for backward transformations, algorithm for edge-renaming is used, and no modification on the target is actually given. However, we suppose presence of modification would not make much difference in the running time. Running time of forward transformation in which rewriting is applied (last two columns) includes time for rewriting. Rewriting took 0.006 CPU seconds at the worst case (*PIM2PSM*, ours). *Class2RDB* stands for class diagram to table diagram transformation, *PIM2PSM* for platform independent model to platform specific model transformation, *C2Osel* is for transformation of customer oriented database into order oriented database, followed by a simple selection, and *UnQL* is the example that is extracted from our previous paper [8], which was borrowed from [4]. It is a composition of two **recs**.

The numbers in parentheses show how often the fusion transformation happened. For example, *PIM2PSM* led to 3 fusions based on the second rule, and further enhanced rewriting led to 10 more fusion rule applications, all of which promoted to the first rule via proposed rewriting rule (2). Same promotions happened to *C2Osel*. Except for *C2Osel'*, a run-time optimization in which unreachable parts are removed after every application of **rec** is applied. Enhanced rewriting led to performance improvements in both forward and backward evaluations, except *C2Osel*. Comparing “previous” with “no rewriting”, *PIM2PSM* and *C2Osel'* led to slowdown. These slowdown are explained as follows. The fusion turns composition of **recs** to their nesting. In the presence of the run-

time optimization, composition is advantageous than nesting when only small part of the result is passed to the subsequent **recs**, which will run faster than when passed entire results (including unreachable parts). Once nested, intermediate result is not produced, but the run-time optimization is suppressed because every execution of the inner **rec** traverses the input graph. *C2Osel'* in which run-time optimization is turned off, shows that the enhanced rewriting itself lead to performance improvements.

## 6 Related Work

Some optimization rules were mentioned in [8], but relationship with static marker analysis was not covered in depth. By enhanced marker analysis and rewriting rules in present paper, all the rules in [8] can be generalized uniformly.

An implementation of rewriting optimizations was reported in [6] but concrete strategies were not included in the paper.

Full (technical report) version of [3] dealt with plugging constructor-only expressions into output marker expressions. It was motivated by authors need to express semantics of @ at the constructor expression level and not graph data level as in [4]. It also mentioned renaming of markers to avoid capture of the output markers in the cycle expressions. We do attempt the same thing at the expression level but we argue here more formally.

The technical report also mentioned the semantics of **rec** on the cycle constructor expressions, even when the body expressions refer to graph variables, although marker environment that maps markers to connected subgraphs introduced there makes the semantics complex. The journal version [4] did not include this semantics on the cycle constructor expressions. But we could use the semantics to enhance rewriting rules for **rec** with **cycle** arguments.

The journal version mentioned run-time evaluation strategy in which only necessary components of structural recursion is executed. For example, only  $\&z_1$  component of **rec** in  $\&z_1 @ \mathbf{rec}(-)(-)$  is evaluated.

A static analysis of UnCAL was described in [1], but the main motivation was to analyze structure of graphs using graph schema.

## 7 Conclusion

In this paper, under the context of graph transformation using UnCAL graph algebra, enhanced static marker inference is first formalized. Fusion rule becomes more powerful thanks to the static marker analysis. Further rewriting rules based on this inference are also explored. Marker renaming for capture avoidance is formalized to support the rewriting rules.

Preliminary performance evaluation shows the usefulness of the optimization for various non-trivial transformations in the field of software engineering research.

Under the context of bidirectional graph transformations [6], one of the advantage of static analysis is that we can keep implementation of bidirectional

interpreter intact. Future work under this context includes reasoning about effects on the backward updatability. Although rewriting is sound with respect to well-behavedness of bidirectional transformations, backward transformation before and after rewriting may accept different update operations. Our conjecture is that simplified transformation accepts more updates, but this argument requires further discussions.

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